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Branes at orbifolds versus Hanany-Witten in six dimensions

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Abstract: We reconstruct non-trivial 6d theories obtained by Blum and Intriligator by considering IIB or $SO(32)$ 5 branes at ALE spaces in the language of Hanany-Witten setups. Using ST duality we make the equivalence of the two approaches manifest, thereby uncovering several new T-duality relations between the group theoretic data describing the embedding of the instantonic 5 brane in the ALE and brane positions in the Hanany-Witten language. We construct several new 6d theories, which can be understood as arising on 5 branes in IIB orientifolds with oppositely charged orientifold planes recently introduced by Witten.

Keywords: Field Theories in Higher Dimensions, Branes in String Theory, Anomalies in Field and String Theories.

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Contents

1 Introduction

The past few years have seen tremendous progress in our understanding of field and string theories. Recent interest has focused on trying to understand the interconnection between the various results in field and string theories. One of the tools used in these studies is the realization of Super-Yang-Mills theories as the low-energy theory on the worldvolume of D-branes. Many strange field theory phenomena like dualities and non-trivial fixed points this way find there natural place in string theory.

Contact between D-brane worldvolumes and stringtheory has been made by using 'branes as probes' [1, 2]. One studies the worldvolume of a Dp brane in a given geometric background. Thereby one engineers a p+1 dimensional field theory. More recently Hanany and Witten provided another brane setup [3] where interesting dynamics on the worldvolume is engineered by suspending the Dp branes between two NS branes in the background of various other branes. Since in this setup one worldvolume direction is compact, we are effectively dealing with a p dimensional SYM theory at low energies.

We'd like to show the relation between these two approaches in the case of non-trivial fixed points with $N = 1$ in 6 dimensions. The most prominent example of this kind of fixed point is the small E_8 instanton [4]. But soon after its discovery it became clear that there are more of these fixed points. Their existence was conjectured from gauge theory considerations [2] and also constructed on 5 brane probes at orientifolds [2]. Many were explicitly constructed via F-theory on Calabi-Yau surfaces [5]. In [6] even more new 6d fixed points were constructed by considering 5 brane probes at ADE orbifolds. They arise as strong coupling limits of certain product gauge groups.

In [7] a method was explained to obtain several of these fixed points from a simple Hanany-Witten like setup involving 6 branes and 5 branes. The setup was generalized in [8] by the inclusion of 8 branes. In this paper we expand the method of [7] to describe the strong coupling fixed points of product gauge groups. We show the relation between this approach and the '5 brane at orbifolds' approach of [6], thereby uncovering some interesting new T-duality relations between brane positions and instanton data. In addition we find some new fixed points which were not realized in a stringy setup before. They also have an interpretation as '5 branes at orbifolds' once one allows for orientifolds of IIB theory with oppositely charged orientifold planes like they were considered recently in [9].

With the same setups one can also construct 'little string theories' with $N = 1$ in the spirit of [10] .

In section 2 we will review the method of [7] to construct $N = 1$ 6d fixed points from a Hanany-Witten setup. Section 3 contains a summary of the results of [6], where the strong coupling fixed points of certain product gauge groups associated to Dynkin diagrams are constructed via type IIB and heterotic 5 branes at orbifolds. In section 4 we construct the same fixed points via the Hanany-Witten configuration. We argue that the 2 approaches are in fact closely related. They can be mapped into each other via an ST duality transformation. By matching the obtained fixed point theories we uncover some interesting new T-duality mappings between the group theoretic data describing the embedding of the instantonic 5 brane in the ALE space and the position of branes.[1]

In section 5 we use this method to construct new fixed point theories and hence also new little string theories. They can be obtained in the TS dual language as wordvolume theories of 5 branes in IIB orientifold theories with oppositely charged orientifold planes introduced recently in [9].

2 6d fixed points from Hanany-Witten setups

The original Hanany-Witten [3] setup yields an $N = 4$ supersymmetric theory in three

[1]This analysis is in a similar spirit to the one from [11] where ST duality maps the data describing monopoles on ALF spaces to brane positions in the Hanany-Witten language

dimensions. Its basic ingredients are IIB NS 5 branes, D5 branes and D3 branes. The worldvolumes of the various branes occupy the following directions:

	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
NS 5	x	x	x	x	x	x	o	o	o	o
D 5	x	x	x	o	o	o	o	x	x	x
D 3	x	x	x	o	o	o	x	o	o	o

In the x^6 direction the 3 branes are suspended between 5 branes. The effective low energy theory is the 2+1 dimensional field theory living on the extended 3 brane directions. Fluctuations of the larger branes are considered much heavier than those of the 3 branes. Moving around the higher dimensional branes hence corresponds to changing parameters of the low energy theory, moving around the 3 branes corresponds to changing moduli of the theory. The idea behind this approach can be summarized as the statement, that the low energy dynamic is determined by the lowest dimensional brane in the setup. The number N_c of 3 branes determines the gauge group, which is $U(N_c)$ in the standard setup [2] , or $SO(N_c)$ and $Sp(N_c)$ [3] upon including orientifolds. Hence we will refer to them as color branes. Similarly the D5 act as flavor branes since they introduce massless hypermultiplets. Another way to add matter is to add semi-infinite 3 branes.

In [7] a similar setup was proposed to describe 6d gauge theories with $N = 1$ supersymmetry and their strong coupling fixed points. Roughly speaking one applies 3 T-dualities to the Hanany-Witten setup, yielding IIA 6 branes suspended between NS 5 branes. The flavor branes in this setup are D8 branes. These D8 branes only appear once one allows a cosmological constant [8] and hence considers a massive IIA [13] configuration.

2.1 The basic brane configuration

After applying T-dualities on the 3, 4 and 5 directions we obtain IIA and the following setup of branes: two NS 5 branes as usual along the 012345 coordinates. Stretched between them N_c D6 branes with a worldvolume along the 0123456 coordinates. In addition there are n_l semi-infinite 6 branes ending on the left NS brane and n_r semi-infinite 6 branes ending on the right. We will again call them the $N_f = n_l + n_r$ flavor branes. The inclusion of D8 branes following the proposal of [8] will be discussed later on. Figure 1 shows the basic brane setup for an $SU(N)$ gauge group with $2N$ flavors. Note that in order to get the low energy field theory corresponding to this brane setup we should look for the lowest dimensional objects. These are as usual the finite color branes, but this time we get an equally important contribution from the full NS branes.

[2] For 4 and higher dimensions the $U(1)$ part of the $U(N_c)$ expected on coinciding branes is projected out [12]

[3] $Sp(N_c)$ denotes for us the group with N_c dimensional representation. Hence in this case N_c has to be even

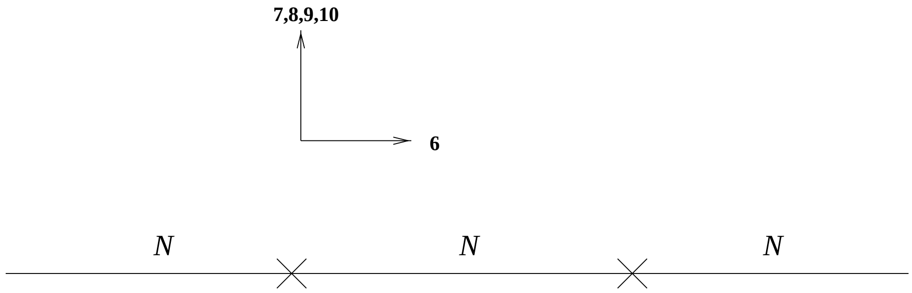

Figure 1: The brane configuration under consideration, giving rise to a 6 dimensional field theory. Horizontal lines represent D6 branes, the crosses represent NS 5 branes.

As shown in [7] every 5 brane contributes one $N = 1$ tensor multiplet. These tensors come from the (2,0) tensor multiplet on the NS5 brane, which decomposes to an (1,0) tensor and a (1,0) hyper. The hyper corresponds to motions of the 5 brane away from the D6 branes and hence is frozen out from the low energy dynamics [7]. One of those tensor multiplets describes the center of mass motion and hence decouples.

2.2 Ramond charge conservation versus anomalies

A D6 brane ending on a NS5 brane is a source of RR 7-form charge. In lower dimensional Hanany-Witten setups the corresponding flux is usually absorbed by fields living on the NS5 brane, leading to a bending of the 5 brane in the worldvolume directions transverse to the end of the color brane. Since here the whole 5 brane is the end of the D6 brane we find that the brane configuration is consistent if and only if the total RR flux on a given NS brane vanishes. With the elements discussed so far this is only possible if we have the same number of D6 branes ending from the left and from the right on a given NS brane, like it is the case in the $SU(N)$ gauge theory with $2N$ flavors from figure 1. Similar one can find consistent SO and Sp gauge theories by introducing orientifolds in the picture.

From the 6d field theory point of view this consistency condition turns up as an anomaly. The anomaly arising from vector and hypermultiplets is

$$I = \alpha \mathrm{tr} F^4 + c(\mathrm{tr} F^2)^2 \,, \tag{2.1}$$

where tr is the trace in the fundamental representation. In the case $\alpha = 0, c > 0$ the anomaly can be cancelled by introducing a tensor multiplet. These anomaly polynomials were analyzed systematically by [14]. It turns out that the only anomaly free theories involving only fundamental matter are those seen in the brane picture. [4]. In addition there are anomaly free $SU(N)$ theories with a symmetric tensor and $N-8$ fundamentals or an antisymmetric tensor and $N+8$ fundamentals. Their brane realization will be discussed in section 4.

[4] There are some subtleties for $SU(2)$ and $SU(3)$ They have no independent 4th order Casimir. There α vanishes automatically and we only get an upper bound on the number of flavors. However global anomalies [5] in these cases restrict us to $N_f = 4, 10$ for $SU(2)$ and $N_f = 0, 6, 12$ for $SU(3)$. The 10 flavor case for $SU(2)$ can be realized by viewing $SU(2)$ as $Sp(2)$ and include orientifolds.

The effective coupling of the gauge theory is

$$\frac{1}{g_{eff}^2} = \frac{1}{g_0^2} + \sqrt{c}\Phi$$

where Φ denotes the vev of the scalar in the tensor multiplet. This is a consequence of coupling the tensor to the gauge fields in order to cancel the anomaly. By redefining the origin of Φ we can write this as

$$\frac{1}{g_{eff}^2} = \sqrt{c}\Phi.$$

In the brane picture this effective coupling corresponds to the distance between the two 5 branes. For $\Phi = 0$, that is when the 5 branes coincide, we get a strong coupling fixed point [2].

It is clear that by repeating the same picture by adding more 5 branes one creates anomaly free product gauge groups, similar like in 4d [15]. Every new 5 brane introduces another $SU(N)$ gauge factor, a new tensor multiplet and a bifundamental hyper multiplet. By tuning the scalars in all these tensor multiplets to bring all the 5 branes together in one point each of these product gauge groups exhibits a non-trivial fixed point.

2.3 Inclusion of 8 branes

As discusses above, the general HW setup contains in addition to the color branes also flavor branes that yield an alternative way of realizing fundamental hypermultiplets (so far we included hypermultiplets by semi-infinite color branes). In our case these flavor branes are D8 branes, as introduced by [8]. This is problematic, since D8 branes are not a solution of standard IIA theory but require massive IIA [13], that is the inclusion of a cosmological constant m. The D8 branes that act as domain walls between regions of space with different values of m. m is quantized and in appropriate normalization [8] can be chosen to be an integer.

The presence of m makes itself known to the brane configuration via a coupling

$$-m \int dx^{10} B \wedge *F^{(8)}, \tag{2.2}$$

where B is the 2 form NS gauge field under which the NS5 brane is charged and F is the 8 form field strength for the 7 form gauge field under which the D6 is charged. From the Bianchi identity for the 2 form field strength dual to $F^{(8)}$ in the presence of a D6 brane ending on a 5 brane, $dF^{(2)} = d * F^{(8)} = \theta(x^6)\delta^{(789)} - mH$ (where $H = dB$), one gets $mdH = \delta^{(6789)}$. The δ function comes from the charge of the D6 brane-end. From here we get back the old result, that for $m = 0$ D6 branes always have to end on a given NS5 brane in pairs of opposite charge (that is from opposite sides). However for arbitrary m this relation shows that RR charge conservation requires that we have a difference of m between the number of branes ending from the left and from the right. [5]

[5] Throughout the paper we will use the following conventions to fix the signs: by passing through an D8 from left to right m increases by 1 unit. In a background of a given m the number of D6 branes ending on a given NS5 from the left is by m bigger than the number of D6 branes ending from the right.

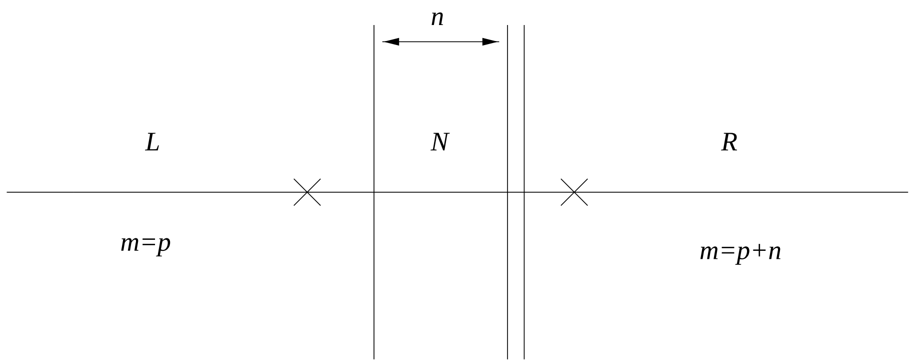

Figure 2: Basic Hanany-Zaffaroni setup.

Figure 2 shows the basic brane configuration involving D8 branes. The n D8 branes in the middle give rise to 8 flavors for the $SU(N)$ gauge group. In addition they raise the cosmological constant m from p to $p+n$. In the background of this values of m the modified RR charge conservation tells us

$$\begin{aligned} N &= L - p \\ R &= N - (p+n) = L - 2p - n \end{aligned}$$

With the $R+L$ flavors from the semi-infinite D6 branes the total number of flavors is $L+R+n = 2N$ in agreement with the gauge anomaly considerations on the 6 brane [14] for every possible value of p.

2.4 Orientifolds

To obtain SO or Sp groups we have to introduce orientifold planes as in [16, 17, 18, 19]. We have in principle 2 possibilities: O8 branes and O6 branes. Let us first consider the O8. We have to distinguish two possibilities: O8 planes with negative or positive D8 brane charge (that is -16 or +16) [20]. The former are the T-dual of the O9 projecting IIB to type I. On the D8 worldvolume they project the symmetry group to an (global) SO group, while on the D6 we get a local Sp group. The positively charged O8 projects on global Sp and local SO. If we want to have vanishing total D8 charge, we should restrict ourselves to either 2 negatively charged O8s with 32 D8 branes or one O8 of each type. Latter configuration was recently studied by Witten [9].

Since these O8 planes carry D8 brane charge, they also effect the cosmological constant like -16 (+16) D8 branes would.

Building product gauge groups from these we have the four possibilities.

The gauge groups corresponding to brane setups in figure 3 have been analyzed in the equivalent setup for 4d in [21]. Their result is indicated in figure 3. If the O8 is in between two 5 branes the 'center' gauge group is projected to $Sp(K)$ or $SO(K)$ respectively. All other gauge groups stay SU, however the SU groups to the right are identified with those to the left and one effectively projects out half of the SU groups. In addition to this, in 6d we got the special situation that the O8 also changes the cosmological constant.

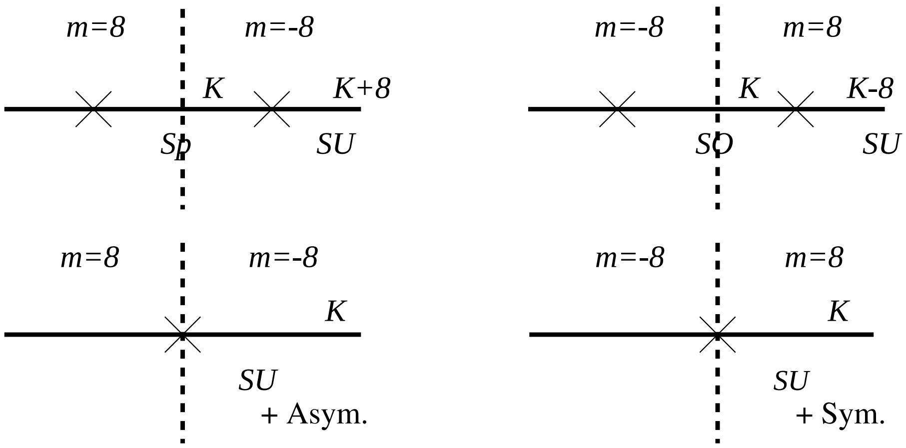

Figure 3: Various possibilities to introduce O8 planes.

For symmetry reasons we have to choose $m = \pm 8$ on the two sides of the orientifold. Therefore we obtain in total

$$Sp(k) \times \prod_i SU(k+8i)$$

or

$$SO(k) \times \prod_i SU(k-8i)$$

respectively. From strings stretching in between neighbouring gauge groups we get bifundamentals $(\Box, \Box)$.

The other possible situation considered in [21] is that one of the 5 branes is stuck to the O8. In this case all the groups stay SU, again the left and the right ones identified leading to effectively half the number of gauge groups. In addition the middle gauge group has a matter multiplet in the antisymmetric or symmetric representation for positive/negative charge O8 planes. In 6d we again have the effect of the cosmological constant and as a result get

$$\prod_i SU(k \pm 8i)$$

with antisymmetric/symmetric tensor matter in the first gauge group factor in addition to the bifundamentals. Note that these gauge theories are all anomaly free [14] upon coupling to the tensor multiplets associated to the independent motions of the 5 branes.

The other realization of SO and Sp groups is to introduce an O6 along the D6 branes. Here the discussion in 6d is analog to the original one in 4d with O4 orientifolds [17, 18] and was performed in [7]. Upon building product gauge groups we get an alternating series of SO and Sp groups with bifundamentals like in 4d [22]. As in 4d this is due to the fact that the orientifold changes sign upon passing through the NS5 brane [17]. It therefore also contributes to the RR charge conservation condition. We obtain a

$$\prod_i SO(N+8) \times Sp(N)$$

chain with bifundamentals.

3 6d fixed points from 5 branes at orbifolds

In this section we will review the analysis of [6].

3.1 $U(N)$ ALE instantons and IIB NS5 branes at orbifolds

6d fixed points and 'little string theories' with (1,1) or (2,0) supersymmetry arise as worldvolume theories on an NS5 brane of IIB and in the 5+1 dimensional space transverse to an ADE singularity respectively, that is one looks at IIB on $\mathbb{C}^2/\Gamma_G$ where Γ_G is a discrete $SU(2)$ subgroup with ADE classification.

The main idea of [27, 6, 23] was to combine these and study NS5 branes at ADE singularities to obtain new $N = 1$ (that is (1,0)) supersymmetric theories. One can determine the gauge group corresponding to this setup in the spirit of the analysis of [24] of instantons on ALE spaces [6] to be

$$\prod_{\mu=0}^{r} U(Kn_\mu)\,, \tag{3.1}$$

where $\mu = 0 \ldots r$ =rank(G) labels the nodes of the extended Dynkin diagram of the ADE group G corresponding to the irreducible representations R_μ of Γ_G, with $|R_\mu| = n_\mu$, the Dynkin indices. The matter multiplets transform as $\frac{1}{2} \oplus_{\mu\nu} a_{\mu\nu}(\Box_\mu, \overline{\Box_\nu})$, where $a_{\mu\nu}$ contains the information about the links in the extended Dynkin diagram, that is $a_{\mu\nu}$ is one if nodes μ and ν are linked in the extended Dynkin diagram and zero otherwise. In addition, there are r $\mathcal{N} = (1,0)$ hypermultiplets and r $\mathcal{N} = (1,0)$ tensor multiplets (which combine into r $\mathcal{N} = (2,0)$ matter multiplets), coming from reducing the 10d two-form and four-form potentials down to 6d on the r cycles which generate H_2 of $\mathbb{C}^2/\Gamma_G$. This theory is anomaly free. The deadly F^4 term vanishes and the remaining gauge anomaly is cancelled by the coupling to the tensors.

The diagonal $U(1)$ factor in $\prod_{\mu=0}^{r} U(Kn_\mu)$ has no charged matter and decouples. The other r $U(1)$ factors do have charged matter and are thus anomalous. This anomaly is cancelled [6] with help of the r hypermultiplets from above, which correspond to the blowing up modes of the $\mathbb{C}^2/\Gamma_G$ singularity. They pair up with the $U(1)$ gauge fields to give them a mass and their expectation values effectively become Fayet-Iliopoulos parameters in the gauge group. The cancellation of the $U(1)$ anomalies using the r hyper-multiplets and the cancellation of the F^2 anomalies using the r tensor multiplets can be regarded as being related by a remnant of the larger supersymmetry of the full type IIB theory.

Following [2], there can be a non-trivial 6d RG fixed point at the origin of the Coulomb branch provided all $g^{-2}_{\mu,eff}(\Phi)$ (Φ denotes the scalar in the tensor multiplet) are non-negative along some entire "Coulomb wedge". However [6] showed that there is always

[6] K p branes inside N p+4 branes can be interpreted as K $U(N)$ instantons [25]. The gauge group on the p branes is given by the Hyper Kahler quotient construction of instanton moduli spaces by Kronheimer [26]. Since IIB does not allow for 9-branes, we have $N = 0$ and hence the interpretation in terms of instantons breaks down. Nevertheless the analysis of the gauge theory on the 5 brane goes through even in the $N = 0$ case.

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one subgroup for which $g_{\mu,eff}^{-2}$ must become negative for large Φ. As already pointed out in [27], one can always take the $U(K)$ corresponding to the extended Dynkin node $\mu = 0$ to be the IR free theory, which means that this gauge group is un-gauged in the IR limit and hence becomes a global symmetry. It is then possible to choose a Coulomb wedge so that the remaining gauge groups in (3.1) all have $g_{\mu,eff}^{-2}(\Phi) \geq 0$ along the entire wedge even in the $g_{\mu,cl}^{-2} \to 0$ limit.

With this the result of [6] is that for any $\mathbb{C}^2/\Gamma_G$, and for every K, these theories give 6d non-trivial RG fixed points with r tensor multiplets and gauge group $\prod_{\mu=1}^{r} SU(Kn_\mu)$, where the $U(1)$s in (3.1) have been eliminated, as discussed above, by the anomaly cancellation mechanism, and the $\mu = 0$ node gives a global rather than local $SU(K)$ symmetry.

If one is considering the 'little string theories' positivity of $g_{\mu,eff}^{-2}$ is not required. Negative $g_{\mu,eff}^{-2}$ just means that the theory hits a Landau pole and hence needs new UV degrees of freedom to be well defined. These additional degrees of freedom are provided by the 'little string'. Hence all $SU(K)$ factors stay interacting. The $U(1)$s and the additional hypermultiplets still drop out via the anomaly cancellation mechanism.

3.2 $SO(32)$ ALE instantons

Another way to obtain new $N = 1$ theories is to study heterotic $SO(32)$ or type I 5 branes at ADE singularities. This was also done by [6]. We can view these theories as IIB orientifolds with one O9 and 32 D9 branes. As mentioned above, K 5 branes inside N 9 branes can be interpreted as K $U(N)$ instantons, and with the the O9 orientifold K 5 branes are K $SO(N)$ instantons. The corresponding gauge group is again obtained via the Hyper Kahler quotient construction [26, 24]. While this construction works for arbitrary N the gauge theory on the 5 brane is anomalous unless N takes the physical values $N = 0$ for $U(N)$ (that is the pure type IIB we discussed in the previous subsection) and $N = 32$ for $SO(N)$. In addition one has to trade 29 hypermultiplets of the Hyper Kahler construction for each of the tensor multiplets that appear in the gauge theory: while the Higgs branch of the theory is supposedly equivalent to the moduli space of instantons given by the Hyper Kahler quotient, the gauge theory on the worldvolume is actually the Coulomb branch of the small instanton. Making this transition exactly trades the 29 hypers into 1 tensor.

Putting the 5 branes at the ADE singularity means that we are dealing with $SO(32)$ instantons on an ALE space. Such instantons are not only characterized by K, but there can also be non-trivial Wilson lines at infinity [6]. An instanton gauge connection is asymptotically flat and thus usually trivial since the asymptotic space X_∞ surrounding the instanton usually has trivial π_1. However, on the ALE space, $\pi_1(X_\infty) = \Gamma_G$ and thus there can be non-trivial Wilson lines at infinity, leading to non-trivial group elements $\rho_\infty \in SO(32)$, representing Γ_G in the gauge group. Thus, in addition to K, the instanton is topologically characterized by integers w_μ in $\rho_\infty = \oplus_\mu w_\mu R_\mu$, giving the representation of Γ_G in $SO(32)$ in terms of the irreps R_μ. In order to have $\rho_\infty \in SO(32)$ we must have $\sum_\mu n_\mu w_\mu = 32$ and $w_\mu = w_{k+1-\mu}$.

In addition there is another discrete choice one can make for ρ_∞, due to the fact that the gauge group of the heterotic string is in fact $Spin(32)/\mathbb{Z}_2$ and not $SO(32)$. The $\mathbb{Z}_2$ is generated by the element w in the center of $Spin(32)$ which acts as -1 on the vector, -1 on the spinor of negative chirality, and $+1$ on the spinor of positive chirality. Because only representations with $w = 1$ are in the $Spin(32)/\mathbb{Z}_2$ string theory, the identity element $e \in \Gamma_G$ can be mapped to either the element 1 or w in $Spin(32)$. Using the terminology of [27] we will refer to this as the case with or without vector structure. [7] The integers w_μ and the distinction of with or without vector structure will have an interpretation in terms of brane positions in the dual HW setup.

3.2.1 The case with vector structure

Consider the worldvolume theory of K type I 5 branes interpreted as instantons on an ALE space. The non-trivial Wilson lines are described by the integers w_μ with $\sum_\mu n_\mu w_\mu = 32$. The analysis for an arbitrary ADE-type singularity was performed in [6]. We will just state their results for the A_k (that is $\Gamma = \mathbb{Z}_{k+1}$) singularity, which already appeared earlier in [27]. For a detailed discussion of this result we refer the reader to [6].

For $k+1$ **even** the gauge group is

$$Sp(V_0) \times \prod_{\mu=1}^{(k-1)/2} SU(V_\mu) \times Sp(V_{(k+1)/2}), \tag{3.2}$$

with $\frac{1}{2}w_0\Box_0$, $\oplus_{\mu=1}^{(k-1)/2} w_\mu \Box_\mu$, $\frac{1}{2}w_{(k+1)/2}\Box_{(k+1)/2}$ and $\oplus_{\mu=1}^{(k+1)/2}(\Box_{\mu-1},\Box_\mu)$ matter multiplets (subscripts label the gauge group). In addition there are $(k+1)/2$ tensor multiplets. The V_μ are given by

$$V_0 = 2K, \quad V_{i\neq 0} = \sum_{j=1}^{k} C^{-1}_{ij}(w_j + D_j),$$

where C^{-1}_{ij} is the inverse $SU(k+1)$ Cartan matrix, given by $C^{-1}_{i<j} = i(k+1-j)/(k+1)$. w_μ are defined above to contain the information about the Wilson lines at infinity. While in the Hyper Kahler quotient construction D_μ is given by $D_\mu = -\delta_{\mu,0}$, anomaly freedom of the gauge theory demands $D_\mu = -16\delta_{\mu,0} - 16\delta_{\mu,(k+1)/2}$. This corresponds precisely to the trading of the hypermultiplets on the Higgs branch to the tensors on the Coulomb branch. With this information one can write the V_μ as

$$V_\mu = 2K + \sum_{\nu=0}^{(k+1)/2} min(\mu,\nu)\cdot W_\nu - 8\mu\,,$$

where $W_0 = \frac{1}{2}w_0$, $W_{(k+1)/2} = \frac{1}{2}w_{(k+1)/2}$ and all other $W_i = w_i$.

[7] One is really only talking about the behaviour of a flat connection at infinity. There can be non-trivial obstructions to vector structure even when the flat connection at infinity is trivial [28]

For $k+1$ **odd** we similarly get

$$Sp(V_0) \times \prod_{\mu=1}^{k/2} SU(V_\mu)\,, \tag{3.3}$$

with $\frac{1}{2}w_0\Box_0$, $\oplus_{\mu=1}^{k/2} w_\mu \Box_\mu$, $\oplus_{\mu=1}^{k/2}(\Box_{\mu-1},\Box_\mu)$ and $\begin{smallmatrix}\Box\\\Box\end{smallmatrix}_{k/2}$ matter multiplets and $k/2$ tensor multiplets. Similar as above the V_μ are given as [8]

$$V_\mu = 2K + \sum_{\nu=0}^{k/2} min(\mu,\nu)\cdot W_\nu - 8\mu\,,$$

where this time only $W_0 = \frac{1}{2}w_0$ and all other $W_i = w_i$. This identification will become more transparent in the HW picture.

3.2.2 The case without vector structure

For the case without vector structure only the A-type singularity was analyzed in [27]. This possibility only exists for $k+1$ even. The result is

$$\prod_{\mu=0}^{(k-1)/2} SU(V_\mu)\,, \tag{3.4}$$

with $\oplus_{\mu=1}^{(k-1)/2} w_\mu\Box_\mu, \oplus_{\mu=0}^{(k-1)/2}(\Box_{\mu-1},\Box_\mu), \begin{smallmatrix}\Box\\\Box\end{smallmatrix}_0$ and $\begin{smallmatrix}\Box\\\Box\end{smallmatrix}_{(k-1)/2}$ matter multiplets and $(k-1)/2$ tensor multiplets. The V_μ are in this case

$$V_\mu = 2K + \sum_{\nu=0}^{(k-1)/2} min(\mu,\nu)\cdot w_\nu - 8\mu\,,$$

where in this case one had to use $D_\mu = -8\delta_{\mu,0} - 8\delta_{\mu,(k-1)/2} - 8\delta_{\mu,(k+1)/2} - 8\delta_{\mu,k}$.

3.3 Theories with twisted 5 branes

Another class of fixed points has been found in [6] by considering what they called twisted 5 branes in type I. Again we will just quote their result for the A_k case. The interpretation of the twisted 5 brane will become clear by considering the dual HW setup. They only exist for even $k+1$. The theory obtained this way for A_k singularity and K NS branes has gauge group

$$\prod_{s=0}^{(k-1)/2} Sp(V_{2s}) \times SO(V_{2s+1})$$

and $\oplus_{\mu=1}^{k+1}(\Box_{\mu-1},\Box_\mu)$ matter multiplets (where the $(k+1)$th gauge group is identified with the 0th). Similar to the other cases

$$V_0 = 2K, \quad V_{i\neq 0} = \sum_{j=1}^{k} C_{ij}^{-1} D_j$$

[8] In this case $D_\mu = -16\delta_{\mu,0} - 8\delta_{\mu,k/2} - 8\delta_{\mu,(k+2)/2}$.

and $D_\mu = (-16, 16, -16, \cdot\cdot, -16, 16)$. It's easy to see that this yields

$$\{Sp(2K) \times SO(2K+8)\}^{(k+1)/2} \ . \tag{3.5}$$

The gauge anomaly is cancelled by k tensors.

4 Relation between the two approaches

The fixed points described in the last section arise as the worldvolume theories of NS or heterotic 5 branes. In the spirit of [29] one would not expect these probes to be good probes of the background geometry. In the limit that we get a decoupled theory on the brane any information about compactness of the background is lost. Clearly the singularity type matters since one obviously gets different theories by considering different singularities. This means that we should see the same picture by considering 5 branes with a transverse ALF space instead of the ALE, that is a space that asymptotically looks like $(\mathbb{R}^3 \times S^1)/\Gamma$ instead of the ALE which looks like $\mathbb{R}^4/\Gamma$ But this configuration can be looked at in a ST dual picture: S-duality takes the IIB NS5 brane into an D5 brane or similar the heterotic 5 brane into an type I 5 brane, which can also be considered a IIB D5 at an orientifold 9 plane with 32 embedded D9. T-duality on the ALF isometry turns D5 branes into D6 branes and an A_k singularity into $k+1$ NS 5 branes. Note that this exactly maps the description of section 3 to that of section 2. This mapping is valid whether one goes to the limit where the theory on the brane decouples as a 'little string' theory [10, 23] or if we also decouple these stringy excitations and study the strong coupling fixed point. However the dual picture of the 'little string' theories necessary involves a limit where the dual radius is taken to zero. Therefore we will mostly limit ourselves in the rest of the discussions to the realization of strong coupling fixed points of the renormalization group. We will explore this correspondence in more detail in the various cases thereby uncovering several interesting T-duality relations.

Note that almost the same duality has been used in [30] to map the QM system describing D0 branes on an ALE space to a Hanany-Witten setup with strings stretching between 5 branes.

4.1 IIB NS5 branes at an A_k singularity

As shown by [27, 6] and reviewed above, the gauge theory on K NS5 branes on a transverse A_k singularity yield an anomaly free $SU(K)^{k+1}$ gauge group with k tensor multiplets and bifundamentals transforming under the ith and $(i+1)$th gauge group, where the $(k+2)$th gauge factor is again the first. Or in the language of [6] we have to use the values of n_μ and $a_{\mu\nu}$ for the A_k extended Dynkin diagram, that is $n_\mu = 1$ for all μ and only $a_{\mu\nu} = \delta_{\mu,\mu+1}$. With this data we obtain

$$\prod_{\mu=0}^{r} U(Kn_\mu) \tag{4.1}$$

gauge group with matter multiplets in representations $\frac{1}{2}\oplus_{\mu\nu} a_{\mu\nu}(\Box_\mu,\overline{\Box}_\nu)$. (4.1) reduces to (3.1) once on takes into account that the additional hypermultiplets and $U(1)$s have been eaten up by the anomaly cancellation mechanism of [6]. In order to go to the non-trivial fixed points we will have to tune all coupling constants to infinity. As mentioned above this is not possible. One gauge factor always becomes IR free, so that at the fixed point we only have an $SU(K)^k$ gauge group. The last $SU(K)$ factor becomes a global symmetry.

The same theory is described in the following HW setup with k NS branes. RR charge conservation at every NS5 brane guarantees anomaly freedom of the gauge

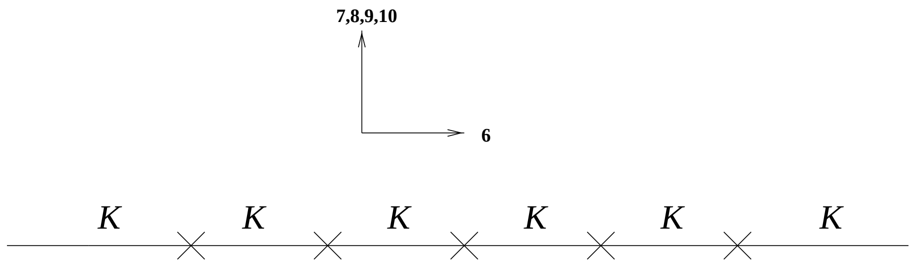

Figure 4: Brane configuration yielding a $SU(K)^k$ gauge group with bifundamentals.

theory. The matter content and gauge group can be simply read off from the brane construction. It agrees with the one from above.

Turning on Fayet-Iliopoulos terms in the HW picture corresponds to moving the 5 branes away in the 7,8,9 (and 10, once we lift to M-theory) direction. This leaves us with one less 5 brane, corresponding, as expected, to blowing up the A_k singularity to an A_{k-1}. As reviewed in section 2 by letting the 6 branes end on the 5 branes in the HW setup we effectively locked the hypermultiplet arising from the decomposition of the (2,0) tensor on the NS5 brane under (1,0) supersymmetry and in addition froze out the $U(1)$ part of the gauge theory on the D6 brane. This is the dual realization of the anomaly cancellation mechanism of [6]: a hypermultiplet and a $U(1)$ vector get massive and turn into an Fayet-Iliopoulos term for the gauge theory, corresponding to a blowing-up mode of the singular space.

To make the duality connection we compactify the 6 direction. Upon TS duality the K 6 branes turn into K NS branes and the $k+1$ NS branes yield the A_k singularity. We hence made the equivalence of the two approaches manifest. For various limits of the string coupling and the radius of the compact dimension one or the other description is more appropriate.

Once we compactified on the circle we see the full $SU(K)^{k+1}$ gauge symmetry. Tuning the moduli in such a way that we obtain the fixed point theory amounts to moving all the 5 branes on top of each other. Since the distance between the 5 branes is the effective coupling constant of the corresponding gauge theory we see that there is always one gauge group who's color branes stretch around the entire circle, no matter where we choose to bring our 5 branes together. This is a nice realization of the fact that there's always one gauge factor with a finite gauge coupling that becomes free in the IR and hence becomes a global symmetry.

A special case of this setup on the circle is if we have only one 5 brane on the circle. In this case the bifundamental becomes just an adjoint. There are no tensor multiplets left. In the field theory in this case the anomaly polynomial is identical zero. In fact the adjoint hypermultiplet combines with the vector multiplet to yield an (1,1) supersymmetric gauge theory. This is of course no big surprise, since on the dual side we are in this case just dealing with the theory of IIB NS5 branes on a non-singular space.

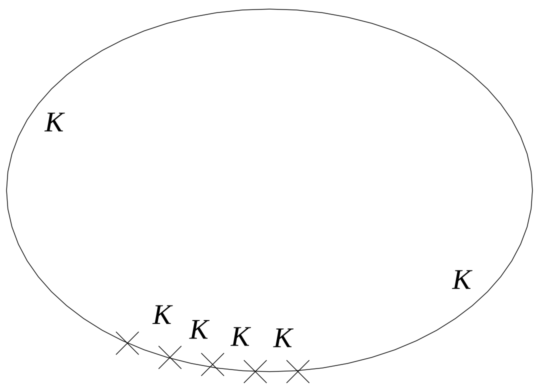

Figure 5: Brane configuration of 4 with x_6 compact.

4.2 $SO(32)$ 5 branes at an A_k singularity

From the discussion of section 3 we recall that according to the analysis of [6] $SO(32)$ 5 branes at an A_k singularity can be characterized by the following data: K, the number of 5 branes, the singularity A_k, the Wilson lines at infinity described by integers w_μ satisfying $\sum_\mu w_\mu = 32$ (μ runs from 0 to k labelling the nodes of the extended Dynkin diagram) and the the discrete choice whether the identity in Γ_{k+1} is mapped to 1 or to w in $Spin(32)/Z_2$, that is to say in the language of [27] whether we are considering the case 'with or without' vector structure.

In what follows we will reproduce the same gauge theories in the language of the Hanany-Witten setup thereby producing a dictionary of how the data describing the theory gets mapped under T-duality. Information about the geometry of the gauge bundle like the integers w_μ and the distinction of 'with or without' vector structure have a nice interpretation just in terms of brane positions.

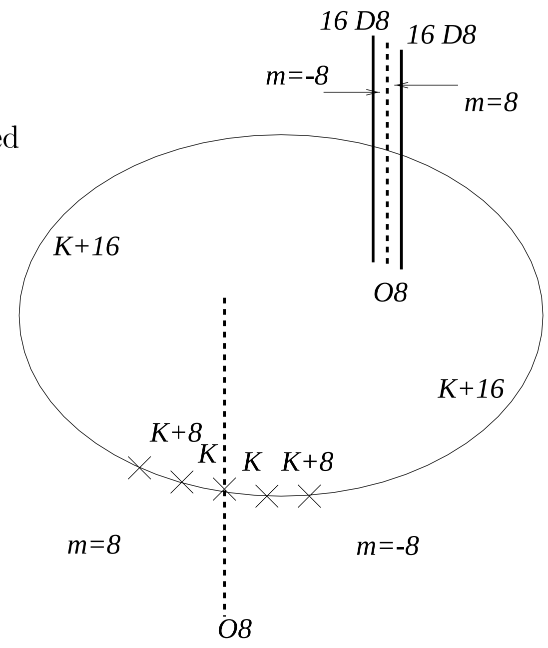

Figure 6: Brane configuration yielding a product of two Sp and several SU gauge groups bifundamentals.

Figure 6 shows the basic HW setup describing this kind of theory. In addition to the $k+1$ NS 5 branes corresponding to the A_k singularity and the D6 branes stretching between them, we introduced two O8 orientifold planes. They are the T-dual picture of the O9 which one introduces to go from IIB to type I. In addition we have 32 D8 branes. Their position on the circle is described by the integers w_μ that contained the data about the instanton embedding in the picture of [6]: w_μ just counts the number of D8 branes between 5 brane number μ

and number $\mu+1$. Similar the D_μ from above which was chosen to have a particular value to cancel the anomaly corresponds to D8 brane charge from the orientifolds. The anomaly free values of D_μ just tell us that the orientifolds are two lumps of D8 brane charge -16 each on symmetric positions between the 5 branes. The condition $w_\mu = w_{k+1-\mu}$ is simply the statement that the construction is symmetric with respect to the orientifolds. If $k+1$ is odd, one of the 5 branes is stuck at one of the orientifold planes. If $k+1$ is even, we can either have all 5 branes free to move around on the circle or we have one 5 brane stuck at each of the two orientifolds. This will correspond to the distinction of 'with or without' vector structure. We will explain this correspondence in more detail by comparing the gauge groups that arise.

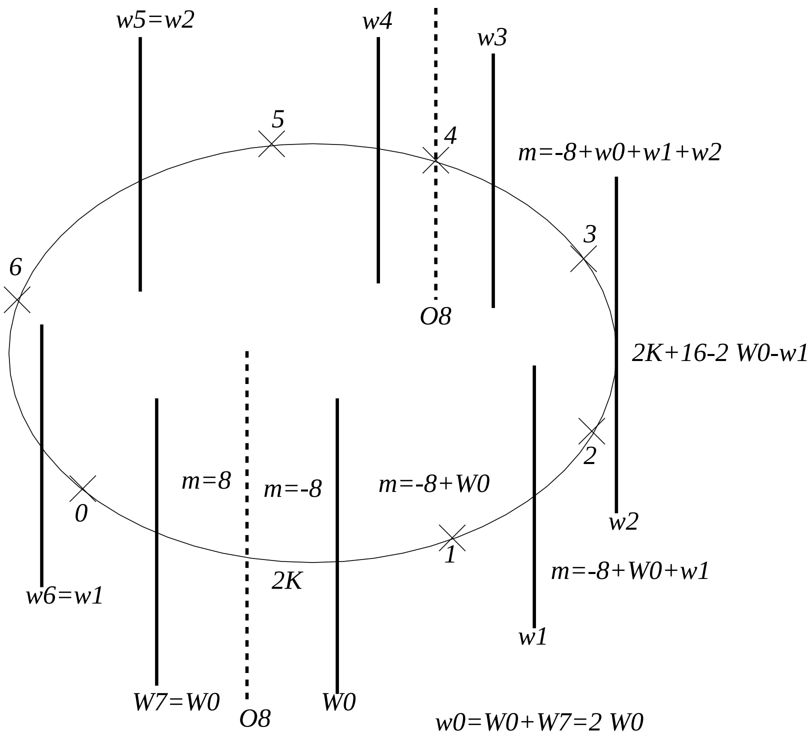

Figure 7: Notation in the example of A_6, that is 7 5 branes on the circle.

Figure 7 illustrates the notation. Again it is useful to introduce W_μs as defined in the previous section to take care of the effect that for every gauge group whose color branes stretch over the orientifold half of the w_μ branes have to be located on either side. Therefore in these cases we define $W_\mu = \frac{1}{2} w_\mu$ while in all other cases $W_\mu = w_\mu$. These special values of μ are 0 and $(k+1)/2$ for even $k+1$ with vector structure, just 0 for $k+1$ odd and there is no such gauge group for $k+1$ even and without vector structure, as can be easily seen from the identification in terms of brane positions given above.

$k+1$ **odd**

We are studying K 5 branes on an A_k singularity on the IIB side. That is we consider $k+1$ 5 branes on the circle with K D6 branes stretching between them in the HW picture.

As in figure 6 we have two orientifolds on the circle, both with negative charge. One of the 5 branes is stuck on the one orientifold. We label the 5 branes with μ running from 0 to k as in figure 7. [9] In addition there are 32 D8 branes. Let again w_μ denote the number of 8 branes in between the μth and $(\mu+1)$th NS brane. Obviously $\sum_\mu w_\mu = 32$. Also symmetry with respect to the orientifold plane requires $w_\mu = w_{k+1-\mu}$. According to the rules of how to introduce 8 branes and orientifolds [10], we find that the gauge group is

$$Sp(V_0) \times \prod_{\mu=1}^{k/2} SU(V_\mu)\,, \tag{4.2}$$

with

$$V_\mu = 2K - \sum_{\nu=0}^{\mu-1} (\mu-\nu) W_\nu + 8\mu\,.$$

We have $\frac{1}{2} w_0 \Box_0, \oplus_{\mu=1}^{k/2} w_\mu \Box_\mu, \oplus_{\mu=1}^{k/2} (\Box_{\mu-1}, \Box_\mu)$ and $\begin{smallmatrix}\Box\\ \Box\end{smallmatrix}_{k/2}$ matter multiplets (subscripts label the gauge group) and $k/2$ tensor multiplets.

Using the symmetry property $W_\mu = w_\mu = w_{k+1-\mu} = W_{k+1-\mu}$, $W_0 = \frac{1}{2} w_0$ the requirement of having a total of 32 D8 branes $\sum_{\nu=0}^{k} w_\nu = 32$ can be rewritten as

$$\sum_{\nu=0}^{k/2} W_\nu = 16$$

which just says that 16 of the 32 D8 branes have to be on either side of the orientifold. Therefore

$$\sum_{\nu=0}^{\mu-1} \mu W_\nu = 16\mu - \mu \sum_{\nu=\mu}^{k/2} W_\nu$$

and (4.2) becomes

$$V_\mu = 2K - 8\mu + \sum_{\nu=0}^{\mu-1} \nu W_\nu + \sum_{\nu=\mu}^{k/2} \mu W_\nu\,.$$

This gauge group and matter content agrees with (3.3). This shows that our identification of w_μ from above, characterizing the Wilson lines at infinity, and the w_μ we just defined in terms of brane positions indeed is correct. If one wants to study the fixed point associated to this theory, one has to bring the 5 branes together on top of each other. In order to do so one has to move 5 branes through 8 branes, whose position is according to the general philosophy a parameter and not a modulus. Note that the gauge group and matter content is not changed by doing so if one is taking into account that whenever a 5 brane crosses a D8 a new 6 brane is created stretching between them according to the usual HW effect [3]. However our identification of the w_μ with 8 brane positions should not be applied to this situation anymore. Since the kth 5 brane is stuck, we move all

[9] Note that in this numbering the stuck 5 brane is number $(k+2)/2$.

[10] The D8 branes have two effects: first they introduce a fundamental hypermultiplet in the gauge group they are sitting in, second they decrease the number of colors in every following gauge group to their right by one per 5 brane in between, since the value of the cosmological constant m changed.

others on top of this one. In the orbifold language this corresponds to choosing the 0th gauge group factor to be the IR free one. Any other choice would indeed lead to another fixed point. However the kth 5 brane can not participate in this and so these other fixed points are already included in the discussion of lower values of k. $k = 0$ reduces to $Sp(2K)$ gauge theory with 16 $\Box$ and 1 $\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$ matter multiplet and no tensors, the theory on K $SO(32)$ heterotic 5 branes in a non-singular space. This is what we expect, since the 1 NS brane from the HW picture becomes a single KK monopole in the IIB picture that reduces to flat space in the decompactification limit.

$k+1$ even with vector structure

If we have A_k with an even number $k+1$ of 5 branes, we have two choices. We can have all 5 branes free to move around on the circle of Figure 6 or we have one 5 brane stuck at each orientifold separately. Here we will discuss the former case and by comparing the resulting gauge theory see that it corresponds to the case with vector structure. Again we have w_μ D8 branes between the μth and $(\mu+1)$th NS5 brane. Similar to above we find perfect agreement with the orbifold analysis (3.2). With no 5 branes we have an $Sp(2K)$ gauge theory with 16 $\Box$ and 1 $\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$ matter multiplet and no tensors, which is again, as expected, just the $SO(32)$ 5 brane.

$k+1$ even without vector structure

As in the previous case we study $k+1$ 5 branes on the circle with two orientifold planes and 32 D8 brane with $k+1$ even. This time one 5 brane is stuck at every one of the 2 O8 planes. The others are free to move around. We find again perfect agreement with the orbifold analysis (3.4). The simplest case with just the two stuck 5 branes has $SU(2K)$ gauge theory with 2 $\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$, 16 $\Box$ matter fields and no tensors. The corresponding gauge theory has no non-trivial fixed point. However it defines a 'little string theory', as pointed out in [23].

4.3 Theories with twisted 5 branes

From the analysis of 4d Hanany-Witten setups it is known that there are two distinct ways to introduce orthogonal or symplectic gauge groups: either by including and O6 along the D6 flavor branes or an O4 along the D4 color branes. The O6 becomes the O8 after T-dualizing to 6 dimensions. This is the case we studied so far. The other possibility is to include an O6 along the D6 branes in our setup.

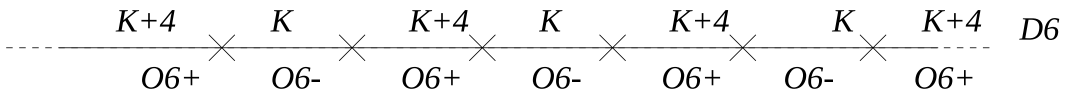

Figure 8: Brane configuration yielding an alternating product of $SO(2K+8)$ and $Sp(2K)$ gauge groups with bifundamentals.

As explained in subection 2.4 this setup shown in figure 8 yields an

$$\{Sp(2K) \times SO(2K+8)\}^{(k+1)/2}$$

gauge group with bifundamentals and k tensors. This can hence be identified with the theories arising from the twisted 5 brane (3.5). The requirement that we have an even number of 5 branes is needed in order to be able to compactify the circle (the gauge group on the far right has to be the same as the one on the far left). This is required for the interpretation of our setup as a little string theory and to be able to go to the dual picture. However from the HW picture we also get consistent gauge theories with an odd number of 5 branes, just yielding another $SO(2K+8)$ or $Sp(2K)$ factor. They probably lead to new non-trivial fixed points.

Now it easy to reformulate this theory in terms of 5 branes at orbifolds. Under T-duality the O4 turns into an IIB O5 plane. Under S-duality the O5 becomes an U5, like for example discussed recently in [31]. The U5 has exactly the same properties with respect to NS5 branes like the O5 for D5 branes. It carries for example charge under the 6-form NS gauge field under which the NS brane is charged. It enhances the (1,1) SUSY gauge theory on the worldvolume of the NS5 brane to SO or Sp respectively. The existence of such an object is required by S-duality. We thus see that the twisted 5 brane of [6] is just an NS5 brane on top of the U5.

5 New fixed points

It is clear that there are several fixpoints that can not be realized via the NS5, D6, D8 and O8 configuration we presented here. They involve all the theories obtained by studying ALE spaces with D or E type singularity. Our approach relied on the fact that upon T-duality in a transverse direction IIB with an A_k type singularity turned into k NS 5 branes. However no statement like this is known for the D and E type singularities. [11] Therefore there are clearly many fixpoints that can only be realized in the picture of branes at orbifolds.

However there are also some examples of theories that can only be constructed in the Hanany-Witten type setup. Once we don't enforce the x_6 to be compact we are basically free to

- put more 8 branes in the picture
- use a different sign for the orientifold projection

If the circle is compact RR charge conservation forces us to have no D8 branes in the case without orientifold and precisely 32 D8 branes once we introduced the O8. In addition we are only allowed to use the O8 with negative D-brane charge in order to be able to cancel its charge with D8 branes. Once we are free to have some surplus of

[11] For IIA on a D-type singularity there is a dual IIB description in terms of NS5 branes on an U5 orientifold plane, the S-dual partner of the O5 [31].

RR charge we are free to use the 'other sign' for the orientifold projection, yielding an O8 with charge +16 and hence SO groups or symmetric tensors instead of Sp groups and antisymmetric tensors like we had so far. In the following we will present some of the theories that can be obtained this way. It was already pointed out in [6] that their anomalies vanish. By embedding them in string theory we show that they actually exist as fixpoint theories. Since it was argued in [33] that 1 uncompact dimension is not enough to let RR flux escape to infinity we should probably limit ourselves to 32 D8 branes or one O8 with charge +16, so that we can cancel the total charge by putting negatively charged O8 branes at infinity.

5.1 Additional D8 branes

The simplest modification is to include additional 8 branes. These theories are basically

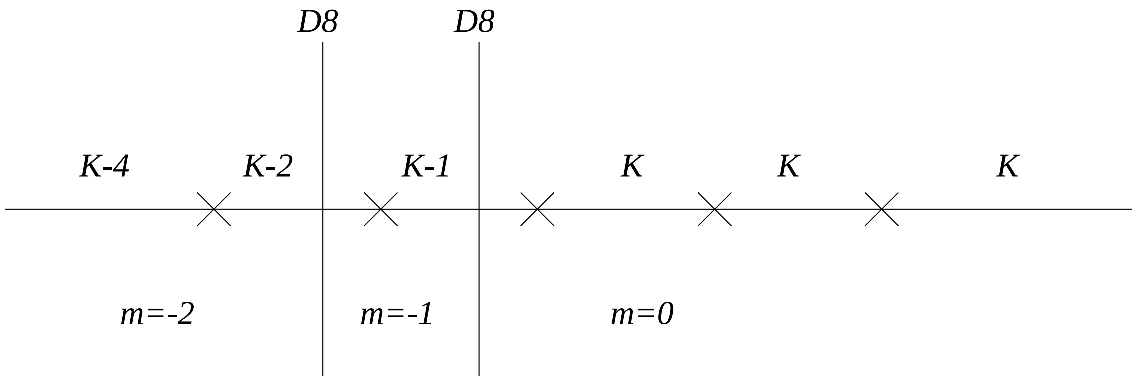

Figure 9: Product gauge group with additional 8 branes.

already obtained in the analysis of subsection 4.2. We start out with the same gauge group on the circle. Going to the fixed point means that we move all the 5 branes together. In subsection 4.2 we put them all on top the O8. We might as well bring them together somewhere else. Since the positions of the 5 branes correspond to the moduli from the tensor multiplets this corresponds to a fixed point at a different point in moduli space. Instead of decoupling one of the Sp groups we decoupled both of them. One interesting example of theories that can be constructed that way is drawn in figure 11. The strong coupling fixed point we obtain from this gauge theory by banging together all the 5 branes can also be obtained by studying an $E_8 \times E_8$ 5 brane at an A_{M-1} singularity [23]. This theory was also analyzed in [34] from F-theory. Unlike the cases discussed above involving $SO(32)$ heterotic 5 branes here we found no obvious duality relation mapping the two descriptions of this fixed points onto each other.

5.2 SO groups and symmetric tensors

As mentioned above we can introduce orientifold O8 planes with positive 8-brane charge in our picture. As mentioned above, we should probably nevertheless insist on having total RR charge zero. This can be done by putting a negatively charged O8 at infinity.

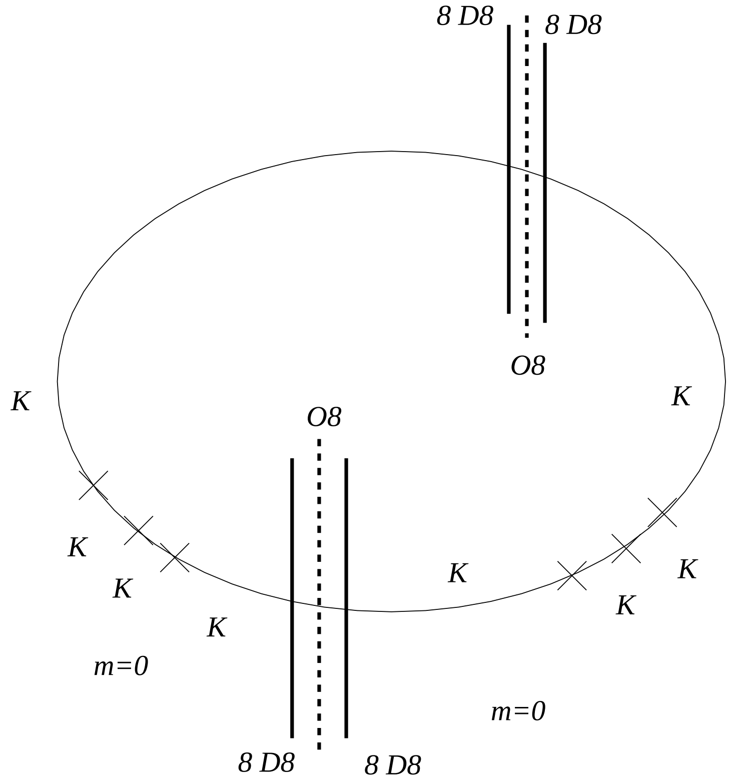

Figure 10: Theory from figure 9 arising from the gauge theory of figure 6 at a different point in moduli space.

This kind of IIA orbifold theory with oppositely charged orientifold planes was recently analyzed in [9]

This way we can generate theories with:

- $SO(2K) \times \prod_{\mu=1}^{(k-1)/2} SU(2K-8\mu) \times \; Sp(2K-4(k+1))$ gauge group and matter content $\oplus_{\mu=1}^{(k+1)/2}(\square_{\mu-1}, \square_\mu)$. In addition there are $(k+1)/2$ tensor multiplets.

- $SO(2K) \times \prod_{\mu=1}^{k/2} SU(2K-8\mu)$ gauge group with $\oplus_{\mu=1}^{k/2}(\square_{\mu-1}, \square_\mu)$ and $\begin{smallmatrix}\square\\\square\end{smallmatrix}_{k/2}$ matter multiplets and $k/2$ tensor multiplets.

- $Sp(2K) \times \prod_{\mu=1}^{k/2} SU(2K+8\mu)$ gauge group with $\oplus_{\mu=1}^{k/2}(\square_{\mu-1}, \square_\mu)$ and $\square\square$ matter multiplets and $k/2$ tensor multiplets.

- $\prod_{\mu=0}^{(k-1)/2} SU(2K+8)$ with $\oplus_{\mu=0}^{(k-1)/2}(\square_{\mu-1}, \square_\mu)$, $\begin{smallmatrix}\square\\\square\end{smallmatrix}_0$ and $\square\square_{(k-1)/2}$ matter multiplets and $(k-1)/2$ tensor multiplets.

These possibilities correspond to an even number of 5 branes free to move on the circle ('with vector structure'), an odd number of branes with one brane stuck on the positively or negatively charged orientifold or an even number of branes with one stuck at each orientifold ('without vector structure'). In these theories the F^4 anomaly vanishes and the F^2 part is cancelled by the coupling to the tensors as above [6, 14] . By embedding

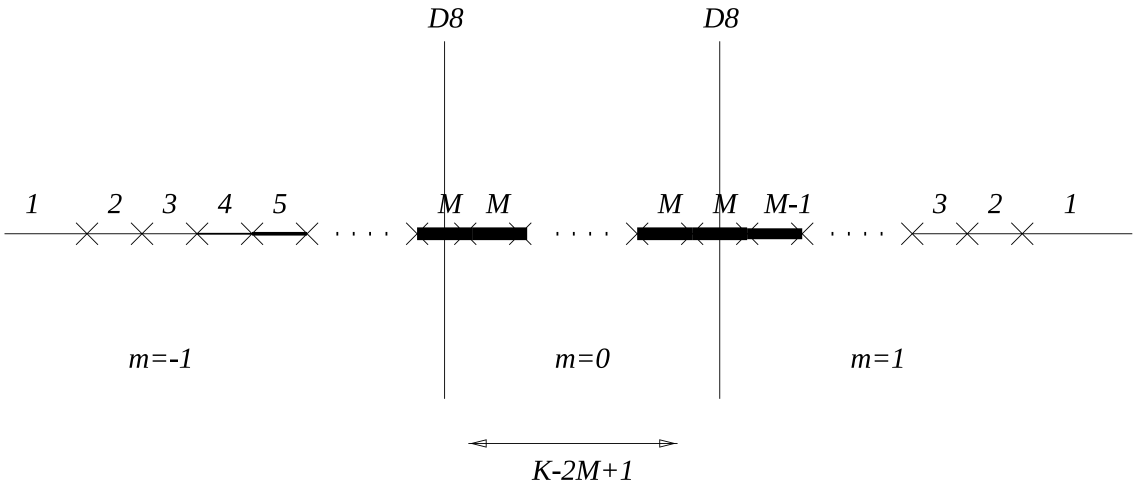

Figure 11: Gauge theory that also appears on an $E_8 \times E_8$ 5 brane at an A_{M-1} singularity.

them into string theory we showed that their corresponding strong coupling fixed points exist.

Witten [9, 32] presented a IIB dual description of this orientifold. There the worldsheet parity operation is combined with a π shift on a compact spacetime circle. In the same spirit we can apply the T-duality used in this paper and thereby find that the fixed points of this section should have a dual realization as the worldvolume theory of NS5 branes in orientifolded IIB on an ALF space, where the worldsheet parity operation is combined with a π shift on the compact ALF circle.

There are three cases when there are no tensor multiplets left in the theory: just one 5 brane stuck to either one of the oppositely charge orientifolds or one stuck to each. The resulting gauge theories are

- $SO(2K)$ with an adjoint

- $Sp(2K)$ with an adjoint

- $SU(2K)$ with an symmetric and an antisymmetric tensor

In all three cases the anomaly vanishes exactly. Even though these theories do not constitute new fixed points they nevertheless define new 'little string theories'.

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References

[1] T. Banks, M. Douglas, N. Seiberg, *Probing F-theory with branes*, *Phys. Lett.* **B 387** (1996) 278 [hep-th/9605199]; *Branes and N=1 duality in string theory*, *Phys. Lett.* **B 400** (1997) 269 [hep-th/9702014].

[2] N. Seiberg, *Non-trivial fixed points of the renormalization group in six dimensions*, *Phys. Lett.* **B 390** (1997) 169 [hep-th/9609161].

[3] A. Hanany and E. Witten, *Type IIB superstrings, BPS monopoles, and three-dimensional gauge dynamics*, *Nucl. Phys.* **B 492** (1997) 152 [hep-th/9611230].

[4] O. Ganor and A. Hanany, *Small E_8 instantons and tensionless non-critical strings*, *Nucl. Phys.* **B 474** (1996) 122 [hep-th/9602120].

[5] M. Bershadsky and C. Vafa, *Global anomalies and geometric engineering of critical theories in six dimensions*, hep-th/9703167.

[6] J. Blum and K. Intriligator, *New phases of string theory and 6d RG fixed points via branes at orbifold singularities*, *Nucl. Phys.* **B 506** (1997) 199 [hep-th/9705044].

[7] I. Brunner and A. Karch, *Branes and six dimensional fixed points*, *Phys. Lett.* **B 409** (1997) 109 [hep-th/9705022].

[8] A. Hanany and A. Zaffaroni, *Chiral symmetry from type IIA branes*, *Nucl. Phys.* **B 509** (1998) 145 [hep-th/9706047].

[9] E. Witten, *Toroidal compactification without vector structure*, hep-th/9712028.

[10] N. Seiberg, *Matrix description of M-theory on T^5 and T^5/Z_2*, *Phys. Lett.* **B 408** (1997) 98 [hep-th/9705221].

[11] S. Cherkis and A. Kapustin, *Singular monopoles and supersymmetric gauge theories in three dimensions*, hep-th/9711145.

[12] E. Witten, *Solutions of four-dimensional field theories via M theory*, *Nucl. Phys.* **B 500** (1997) 3 [hep-th/9703166].

[13] L.J. Romans, *Massive N=2A supergravity in ten-dimensions*, *Phys. Lett.* **B 169** (1986) 374.

[14] U. Danielsson, G Ferretti, J. Kalkkinen and P. Stjernberg, *Notes on supersymmetric gauge theories in five and six dimensions*, *Phys. Lett.* **B 405** (1997) 265 [hep-th/9703098].

[15] J. Brodie and A. Hanany, *Type IIA superstrings, chiral symmetry, and N=1 4D gauge theory dualities*, *Nucl. Phys.* **B 506** (1997) 157 [hep-th/9704043].

JHEP03(1998)003

[16] C. Vafa and B. Zwiebach, *N=1 Dualities of SO and USp Gauge Theories and T-Duality of String Theory*, *Nucl. Phys.* **B 506** (1997) 143 [hep-th/9701015].

[17] N. Evans, C. Johnson and A. Shapere, *Orientifolds, branes, and duality of 4D gauge theories*, *Nucl. Phys.* **B 505** (1997) 251 [hep-th/9703210].

[18] S. Elitzur, A. Giveon, D. Kutasov, E. Rabinovici, A. Schwimmer, *Brane dynamics and N=1 supersymmetric gauge theory*, *Nucl. Phys.* **B 505** (1997) 202 [hep-th/9704104].

[19] A. Brandhuber, J. Sonnenschein, S. Theisen and S. Yankielowicz, *Brane configurations and 4D field theory dualities*, *Nucl. Phys.* **B 502** (1997) 125 [hep-th/9704044].

[20] J. Polchinski, *TASI lectures on D-branes*, hep-th/9611050 and references therein.

[21] K. Landsteiner and E. Lopez, *New curves from branes*, hep-th/9708118.

[22] R. Tatar, *Dualities in 4D theories with product gauge groups from brane configurations*, hep-th/9704198;
C. Ahn and R. Tatar, *Geometry, D-branes and N=1 duality in four dimensions with product gauge group*, *Phys. Lett.* **B 413** (1997) 293 [hep-th/9705106].

[23] K. Intriligator, *New string theories in six dimensions via branes at orbifold singularities*, hep-th/9708117.

[24] M. Douglas and G. Moore, *D-branes, quivers, and ALE instantons*, hep-th/9603167.

[25] M. Douglas, *Branes within branes*, hep-th/9512077.

[26] P. Kronheimer and H. Nakajima, *Math. Ann.* **288** (1990) 263.

[27] K. Intriligator, *RG fixed points in six dimensions via branes at orbifold singularities*, *Nucl. Phys.* **B 496** (1997) 177 [hep-th/9702038].

[28] P. Aspinwall, *Point-like instantons and the* $Spin(32)/IZ_2$ *heterotic string*, *Nucl. Phys.* **B 496** (1997) 149 [hep-th/9612108].

[29] N. Seiberg and S. Sethi, *Comments on Neveu-Schwarz five-branes*, hep-th/9708085.

[30] A. Hanany and G. Lifschitz, *M(atrix) theory on* T^6 *and a m(atrix) theory description of KK monopoles*, hep-th/9708037.

[31] E. Witten, *New "gauge" theories in six dimensions*, **JHEP 01** (1998) 001 [hep-th/9710065].

[32] A. Dabholkar and J. Park, *Strings on orientifolds*, *Nucl. Phys.* **B 477** (1996) 701 []hep-th/9604178.

[33] J. Blum and K. Intriligator, *Consistency conditions for branes at orbifold singularities*, *Nucl. Phys.* **B 506** (1997) 223 [hep-th/9705030].

[34] P. Aspinwall and D. Morrison, *Point-like instantons on K3 orbifolds*, *Nucl. Phys.* **B 503** (1997) 533 [hep-th/9705104].